Course Code: 23HS0833 **R23**

SIDDHARTH INSTITUTE OF ENGINEERING & TECHNOLOGY: PUTTUR
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QUESTION BANK (DESCRIPTIVE)

Subject with Code: Complex Variables & NumericalMethods (23HS0833) **Course & Branch**: B.Tech - EEE

Year & Sem: II-B.Tech & I-Sem **Regulation:** R23

UNIT –I
COMPLEX VARIABLE – DIFFERENTIATION

1	a) Define analytic function.	[L1][CO1]	**[2M]**
	b) State Cauchy-Riemann (C-R) equations in cartesian coordinates.	[L1][CO1]	**[2M]**
	c) Find where the function $w = \frac{1}{z}$ ceases to be analytic.	[L1][CO1]	**[2M]**
	d) Define harmonic function.	[L1][CO1]	**[2M]**
	e) Prove that $f(z) = \bar{z}$ is not an analytic at any point.	[L5][CO1]	**[2M]**
2	a) Show that z^2 is an analytic for all z.	[L2][CO1]	**[5M]**
	b) Determine p such that the function $f(z) = \frac{1}{2}log(x^2+y^2) + i\,tan^{-1}\left(\frac{px}{y}\right)$ is an analytic function.	[L5][CO1]	**[5M]**
3	a) Find whether $f(z) = sinxsiny - icosxcosy$ is an analytic or not.	[L1][CO1]	**[5M]**
	b) Determine whether the function $f(z) = 2xy + i(x^2 - y^2)$ is analytic.	[L5][CO1]	**[5M]**
4	a) Show that $\mathrm{u} = \mathrm{e}^{-\mathrm{x}}(\mathrm{x}\sin\mathrm{y} - \mathrm{ycosy})$ is harmonic function.	[L2][CO1]	**[5M]**
	b) Show that $u = 2\log(x^2+y^2)$ is harmonic function.	[L2][CO1]	**[5M]**
5	Verify that $u = x^2 - y^2 - y$ is harmonic in the whole complex plane and find a conjugate harmonic function v of u?	[L4][CO1]	**[10M]**
6	Prove that, if $u = x^2 - y^2 : v = \frac{-y}{x^2+y^2}$ both u and v satisfy Laplace's equation, but $u + iv$ is not a analytic function.	[L5][CO1]	**[10M]**
7	Show that i) $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4\frac{\partial^2}{\partial z \partial \bar{z}}$. ii) $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\lvert f(z)\rvert^2 = 4\lvert f^1(z)\rvert^2$ where $f(z)$ is an analytic function.	[L1][CO1]	**[10M]**
8	Prove that the function f(z) defined by $f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2}, (z \neq 0)$ *and* $f(z) = 0, (z = 0)$ is continuous and the Cauchy-Riemann equations are satisfied at origin.	[L5][CO1]	**[10M]**
9	Find 'a' and 'b' if $f(z) = (x^2 - 2xy + ay^2) + i(bx^2 - y^2 + 2xy)$ is analytic. Hence find f(z) interms of z.	[L1][CO1]	**[10M]**
10	a) Determine the analytic function whose real part is e^{x}cosy.	[L5][CO1]	**[5M]**
	b) Find the analytic function whose imaginary is $\frac{x-y}{x^2+y^2}$.	[L1][CO1]	**[5M]**
11	a) Find the analytic function f(z) interms of z whose real part is $\mathrm{x}^3 - 3\mathrm{xy}^2$.	[L1][CO1]	**[5M]**
	b) Find the analytic function whose imaginary part is $\mathrm{e}^{\mathrm{x}}(\mathrm{x}\sin\mathrm{y} + \mathrm{ycosy})$.	[L1][CO1]	**[5M]**

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UNIT –II

COMPLEX VARIABLE – INTEGRATION

1	a) Define Line integral	[L1][CO2]	**[2M]**
	b) State Cauchy’s integral theorem.	[L1][CO2]	**[2M]**
	c) State Cauchy Integral formula.	[L1][CO2]	**[2M]**
	d) Expand e^z as Taylor’s series in powers of (z-3).	[L2][CO2]	**[2M]**
	e) State Cauchy Residue theorem.	[L1][CO2]	**[2M]**
2	a) Evaluate $\int_{(0,0)}^{(1,3)} 3x^2ydx + (x^3 - 3y^2)dy$ along the curve $y = 3x$.	[L5][CO2]	**[5M]**
	b) Evaluate $\int_0^{1+i}(x^2 - iy)dz$ along the path $y = x$.	[L5][CO2]	**[5M]**
3	Evaluate $\int_0^{3+i} z^2dz$, i) along the line $y = \frac{x}{3}$ ii) along the parabola $x = 3y^2$.	[L5][CO2]	**[10M]**
4	Show that $\int_c (z+1)dz = 0$ where ‘c’ is the boundary of the square whose vertices at the points $z = 0, z = 1, z = 1+i, z = i$.	[L1][CO2]	**[10M]**
5	a) Evaluate $\int_c \frac{e^{2z}}{(z-1)(z-2)}dz$ where ‘c’ is the circle $\|z\| = 3$.	[L5][CO2]	**[5M]**
	b) Evaluate $\int_c \frac{e^z}{(z-1)(z-4)}dz$ where ‘c’ is $\|z\| = 2$.	[L5][CO2]	**[5M]**
6	Evaluate $\int_c \frac{z-3}{z^2+2z+5}dz$ where ‘c’ is the circle i) $\|z\| = 1$ ii) $\|z+1-i\| = 2$	[L5][CO2]	**[10M]**
7	a) Expand $f(z) = sinz$ in Taylor’s expansion of in powers of $\left(z - \frac{\pi}{4}\right)$.	[L2][CO2]	**[5M]**
	b) Expand $f(z) = \frac{1}{z^2-z-6}$ in Taylor’s series about $i)$ $z = -1$ $ii)$ $z = 1$.	[L2][CO2]	**[5M]**
8	a) Find the Laurent expansion of $\frac{1}{z^2-4z+3}$ for i) $1 < \|z\| < 3$ ii) $\|z\| < 1$.	[L1][CO2]	**[5M]**
	b) Determine the poles of the function i) $\frac{z}{cosz}$ ii) cotz.	[L5][CO2]	**[5M]**
9	a) Find the poles of the function $f(z) = \frac{1}{(z+1)(z+3)}$ and residues at each poles.	[L1][CO2]	**[5M]**
	b) Find the residue of $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ at each pole.	[L1][CO2]	**[5M]**
10	Evaluate $\oint_c \frac{4-3z}{z(z-1)(z-2)}dz$ where ‘c’ is circle $\|z\| = \frac{3}{2}$ using residue theorem.	[L5][CO2]	**[10M]**
11	a) Evaluate $\oint_c \frac{z-3}{z^2+2z+5}dz$ where ‘c’ is circle given by $\|z+1+i\| = 2$ using Residue theorem.	[L5][CO2]	**[5M]**
	b) Evaluate $\int_0^{2\pi}\frac{d\theta}{2+cos\theta}$.	[L5][CO2]	**[5M]**

UNIT –III

SOLUTION OF ALGEBRAIC & TRANSCENDENTAL EQUATIONS

1	a) Find the root of the equation $x^2 - 5 = 0$ by using Bisection method.	[L1][CO3]	**[2M]**
	b) Write the formula to find the root of an equation by Regula Falsi method.	[L1][CO3]	**[2M]**
	c) Write the formula to find the root of an equation by Newton Raphson's method.	[L1][CO3]	**[2M]**
	d) Compare Jacoby and Gauss Seidel methods.	[L5][CO4]	**[2M]**
	e) Solve by Jacoby method [Only two iterations] $x + y = 3\,; 3x - 2y = 4$.	[L3][CO4]	**[2M]**
2	a) Find a positive root of the equation $x^3 - x - 1 = 0$ by Bisection method.	[L1][CO3]	**[5M]**
	b) Find out the square root of 25 given $x_0 = 2.0,\ x_1 = 7.0$ using Bisection method.	[L1][CO3]	**[5M]**
3	a) Find a positive root of the equation $x^4 - x - 10 = 0$ by iteration method.	[L1][CO3]	**[5M]**
	b) Solve $x^3 - 2x - 5 = 0$ for a positive root by iteration method.	[L3][CO3]	**[5M]**
4	Find the root of the equation $x\,e^x = 2$ using Regula-falsi method.	[L1][CO3]	**[10M]**
5	Find the root of the equation $x^3 - x - 4 = 0$ using False position method.	[L1][CO3]	**[10M]**
6	Find a real root of the equation $xtanx+1=0$ using Newton – Raphson method.	[L1][CO3]	**[10M]**
7	Find a real root of the equation $e^x sinx = 1$ using Newton – Raphson method.	[L1][CO3]	**[10M]**
8	Solve the following system of equations by Jacobi method $27x + 6y - z = 85\,; x + y + 54z = 110\,; 6x + 15y + 2z = 72.$	[L3][CO4]	**[10M]**
9	Solve the following system of equations by Jacobi method $2x - 3y + 20z = 25\,; 20x + y - 2z = 17\,; 3x + 20y - z = -18.$	[L3][CO4]	**[10M]**
10	Apply Gauss-Siedel iteration method to solve the equations $20x + y - 2z = 17\,; 3x + 20y - z = -18;\ 2x - 3y + 20z = 25.$	[L3][CO4]	**[10M]**
11	Solve the following system of equations by Gauss-Siedel method $4x + 2y + z = 14\,; x + 5y - z = 10\,; x + y + 8z = 20.$	[L3][CO4]	**[10M]**

UNIT –IV

INTERPOLATION

1

a) Write Newton's forward interpolation formulae. [L1][CO5] **[2M]**

b) Construct a forward difference table for the function y = x^2 for x = 0, 1, 2, 3. [L3][CO5] **[2M]**

c) Write Lagrange's interpolation formulae. [L1][CO5] **[2M]**

d) State the two normal equation used in fitting a straight line. [L1][CO5] **[2M]**

e) Write the normal equations used in fitting a second degree polynomial. [L1][CO5] **[2M]**

2

a) Using Newton's forward interpolation formula and the given table of values [L3][CO5] **[5M]**

x	1	1.4	1.8	2.2
f(x)	3.49	4.82	5.96	6.5

Obtain the value of *f(x)* when *x=1.6*.

b) Applying Newton's forward interpolation formula, compute the value of $\sqrt{5.5}$ given that $\sqrt{5} = 2.236$; $\sqrt{6} = 2.449$; $\sqrt{7} = 2.646$; $\sqrt{8} = 2.828$. [L3][CO5] **[5M]**

3

From the following table values of x and *y=tan x*. Interpolate the values of y when *x=0.12* and *x=0.28.* [L5][CO5] **[10M]**

x	0.10	0.15	0.20	0.25	0.30
y	0.1003	0.1511	0.2027	0.2553	0.3093

4

a) Using Newton's forward interpolation formula and the given table of values [L3][CO5] **[5M]**

x	1.1	1.3	1.5	1.7	1.9
f(x)	0.21	0.69	1.25	1.89	2.61

Obtain the value of *f(x)* when *x=1.4.*

b) Use Newton's backward interpolation formula to find f(32) given f(25)=0.2707, f(30)=0.3027, f(35)=0.3386, f(40)=0.3794. [L3][CO5] **[5M]**

5

Using Lagrange's interpolation formula, find the value of y(10) from the following table: [L3][CO5] **[10M]**

x	5	6	9	11
y	12	13	14	16

6

The values of a function f(x) are given below for certain values of x. Find the values of f(2) using Lagrange's interpolation formula. [L1][CO5] **[10M]**

x	0	1	3	4
y	5	6	50	105

7

By method of least squares fit a straight line to the following data ; [L3][CO5] **[10M]**

X	1	2	3	4	5
y	14	27	40	55	68

8

Fit a straight line $y = a + bx$for the following data [L3][CO5] **[10M]**

X	6	7	7	8	8	8	9	9	10
Y	5	5	4	5	4	3	4	3	3

<table>
<tr><td>9</td><td>Fit a second degree polynomial to the following data by method of least square
<table>
<tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>y</td><td>1</td><td>1.8</td><td>1.3</td><td>2.5</td><td>6.3</td></tr>
</table></td><td>[L3][CO5]</td><td>[10M]</td></tr>
<tr><td>10</td><td>Obtain a second degree polynomial to the data by method of least square
<table>
<tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>Y</td><td>10</td><td>12</td><td>8</td><td>10</td><td>14</td></tr>
</table></td><td>[L3][CO5]</td><td>[10M]</td></tr>
<tr><td>11</td><td>Find the curve of best fit of the type $y = ae^{bx}$to the following data by method of least squares
<table>
<tr><td>X</td><td>1</td><td>5</td><td>7</td><td>9</td><td>12</td></tr>
<tr><td>Y</td><td>10</td><td>15</td><td>12</td><td>15</td><td>21</td></tr>
</table></td><td>[L1][CO5]</td><td>[10M]</td></tr>
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UNIT –V

SOLUTION OF INITIAL VALUE PROBLEMS TO ORDINARY DIFFERENTIAL EQUATIONS

1	a) Write Taylor's formula for $y(x_1)$ to solve $y' = f(x,y)\ with\ y(x_0) = y_0$.	[L1][CO6]	**[2M]**
	b) State Euler formula to solve $y' = f(x,y),\ y(x_0) = y_0\ at\ x = x_0 + h$.	[L1][CO6]	**[2M]**
	c) Find $y^{(1)}(x)$, by Picard's method, given that $\frac{dy}{dx} = 1 + xy\ ;\ y(0) = 1$.	[L1][CO6]	**[2M]**
	d) If $\frac{dy}{dx} = y - x\ ; y(0) = 2, h = 0.2$ then Find the value of k_1 in R–K method of fourth order.	[L1][CO6]	**[2M]**
	e) Write the formula for Runge – Kutta method of fourth order.	[L1][CO6]	**[2M]**
2	Tabulate $y(0.1), y(0.2)\ and\ y(0.3)$ using Taylor's series method given that $y^1 = y^2 + x$ and $y(0) = 1$	[L3][CO6]	**[10M]**
3	Solve $y^1 = x + y$, given $y\ (1) = 0$ find $y(1.1)\ and\ y(1.2)$ by Taylor's series method.	[L3][CO6]	**[10M]**
4	Find an approximate value of $y\ for\ x = 0.1, x = 0.2$ by Picard's method, given that $\frac{dy}{dx} = x + y\ , y(0) = 1$.	[L1][CO6]	**[10M]**
5	Find the values of $y(0.1)\ and\ y(0.2)$ by Picard's method given that $\mathrm{y}^1 = \mathrm{y} - \mathrm{x}^2\ , \mathrm{y}(0) = 1$.	[L1][CO6]	**[10M]**
6	Solve by Euler's method $\frac{dy}{dx} = \frac{2y}{x}$ given y(1)=2 and find $y(2)\ \ and\ y(3)$.	[L3][CO6]	**[10M]**
7	Solve by Euler's method $y' = y^2 + x\ ; y(0) = 1$.and find y(0.1) and y(0.2)	[L3][CO6]	**[10M]**
8	Using modified Euler's method find $y(0.2) and\ y(0.4),\ \ given\ y^1 = y + e^x\ \ , y(0) = 0$	[L3][CO6]	**[10M]**
9	Using Runge – Kutta method of fourth order, compute $y(0.2)$ from $y^1 = xy\ ; y(0) = 1,$ taking $h = 0.2$	[L3][CO6]	**[10M]**
10	Using Runge – Kutta method of fourth order, solve $\frac{dy}{dx} = x^2 - y, y(0) = 1$. Find $(0.1)\ and\ y(0.2)$.	[L3][CO6]	**[10M]**
11	Using Runge – Kutta method of fourth order, find $y(0.1)\ and\ y(0.2)$ given that $\frac{dy}{dx} = x + y, y(0) = 1$.	[L3][CO6]	**[10M]**

Prepared by: Dept. of Mathematics